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INVESTIGATION OF NONLINEAR OSCILLATIONS OF A GAS IN OPEN PIPES

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Results are presented of measurements of velocity and pressure fluctuations in a pipe, open at one end, for nonlinear oscillations, excited by a piston describing harmonic motions, in the linear and nonlinear resonance regions.

It is known that nonlinear oscillations lead to a considerable intensification of various heat and mass transfer processes [1], and this subject has therefore been investigated in many papers [2-24], which have studied the oscillations occurring in pipes. In some of these [2, 3, 16-24] oscillations in closed pipes have been studied, where a piston is moving harmonically at one end, and the other end, the passive end, is closed by a diaphragm. Other papers [2-15] have examined oscillations when the passive end is open to the surrounding medium.

Periodic shock waves [3, 10, 13, 20, 24] can arise both in open and in closed pipes. The amplitude of pressure oscillations in closed pipes is proportional to the power $n = 1/2$ of the piston movement amplitude [2], while opinion varies as to the nature of the relationship in open pipes: some investigators assume $n = 1/3$ [2], while others [6, 7] take $n = 1/2$.

We note that, along with a linear resonance at frequencies

$$\omega_k = \frac{(2k-1)\,\pi a_0}{2L},\ k = 1,\ 2,\ 3,\ \ldots, \tag{1}$$

nonlinear resonances with

$$\omega_k^* = \frac{(2k-1)\,\pi a_0}{4L} \tag{2}$$

may occur in open pipes, their existence being predicted theoretically in [12, 14, 15] and verified experimentally in [13].

The great majority of authors [3, 6, 9, 13, 20] have limited their measurements to pressure oscillations, and only in [5, 7] have attempts been made to measure velocity fluctuations, but these were conducted in the comparatively slight nonlinearity region, where the fluctuations are continuous functions of time. However, it is clear that full information on the oscillations can be obtained only with simultaneous measurement of pressure and velocity.

The present paper has attempted a simultaneous investigation of pressure and velocity fluctuations in open pipes, where shock waves may originate from the open end in the frequency region of linear and nonlinear resonances.

Longitudinal oscillations of the gas column were created in a pipe with one end closed, while at the other end a plane piston was moved according to an harmonic law. In order to maximize the amplitude of oscillations, a compressor was used with a piston stroke of $2l_0$ =

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0.086 m. The piston diameter was $2R_0 = 0.077$ m. The motion of the crankshaft was accomplished via a belt drive from the dc electric motor, which varied the oscillation frequency smoothly. The pipe, of internal diameter $2R = 0.04$ m and length $L_0 = 5.485$ m was made up of 5 parts, so that the experiments could be conducted with various system lengths. The pipe was attached to the compressor cylinder by means of a conical adapter of height h = 0.1 m.

The motor frequency of rotation was measured as follows. During rotation of the pulley, light from an incandescent lamp passed through holes in the pulley and fell on a photoresistor, from which the signal was recorded with a type ChZ-33 frequency meter.

The pressure was measured with a type LKh-610 piezosensor whose signal went to one of the axes of a two-beam type S1-16 electronic oscilloscope. The velocities were measured with a constant temperature thermoanemometer, arranged as in [25]. The thermoanemometer signal went to the second oscilloscope channel. The pressure and the velocity were measured at the same point of the pipe.

It is known [26] that the static and dynamic calibrations of a constant temperature thermoanemometer coincide. Therefore, the anemometer sensor was calibrated in terms of mean velocity.

The oscillograms of the oscillations were recorded photographically. The pressure and velocity oscillograms obtained are complex curves containing discontinuities, and, in addition, the anemometer appears to "rectify" the signal, so that the frequency of the velocity oscillations on the oscillogram turn out to be twice the pressure oscillation frequency. The velocity oscillograms were reduced on the assumption that the average flow velocity is zero. Then, by taking the mirror reflection of half of the period of the velocity oscillations relative to the abscissa axis, one can construct the velocity oscillogram for the whole period. A complete Fourier analysis of the signals would be a laborious process, and, therefore, as in [13], on the oscillograms we measured the so-called scale, i.e., the difference between the maximum and minimum values for the pressure and velocity oscillations. In the linear resonance case one can assume that one half of the scale corresponds, approximately, to the amplitude of the pressure and velocity oscillations. In the other cases one can postulate that there is proportionality between the scale and the amplitude, but no more than that. From the ratio of the scale to the atmospheric pressure one can calculate the dimensionless scale of the pressure fluctuations. The ratio of the velocity fluctuation scale u to the speed of sound in the unperturbed medium a_0 is called the dimensionless velocity scale.

According to [13], to form continuous oscillations one requires that the ratio of the reduced amplitude of motion of the piston

$$l = m^2 l_0, \tag{3}$$

where $m = R_0/R$, to the reduced pipe length L

$$L = L_0 + m^2 l_0 + (m^2 + m + 1)\, h/3 \tag{4}$$

should be more than 0.064. In our case this parameter $\delta = l/L$ has values which are 0.027, 0.032, 0.041, respectively.

Figure 1 shows a series of oscillograms, corresponding to a pipe length $L_0 = 3.485$ m with self-frequency $\omega_1/2\pi = 22$ Hz, at various frequencies of piston fluctuation and at various distances x = 2 m from the end of the pipe. In each frame the upper curve is an oscillogram of the velocity fluctuations, and the lower curve shows the pressure fluctuations. The piston oscillation frequency increases in successive frames, although the frequency of the oscilloscope sweep generator was varied in an arbitrary manner. At frequencies far from the first nonlinear resonance the velocity fluctuations have a symmetric shape, and the pressure fluctuations are weak and discontinuous. In addition, in the pressure drop section there is a positive small pressure peak. With increase of frequency the discontinuity in the velocity drop section becomes noticeable, and the amplitude of the additional pressure peak increases; there is also an increase in the scales of the pressure oscillations (frames 2-3). In addition, a supplementary peak appears in the sections where the velocity is decreasing (frame 2). It is not difficult to see that the velocity scales then decrease, and that the velocity oscillogram has a "saddle" point at the first nonlinear resonance ($\omega/\omega_1 = 0.5$, frame 4), as predicted by Keller [12], with the same value of peak heights. Subsequently,

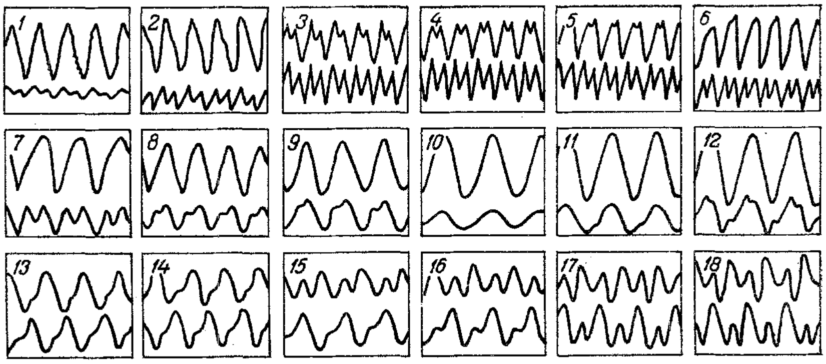

Fig. 1. Oscillograms of the fluctuations of velocity (upper) and of pressure (lower) at distances x = 2 m from the open end of the pipe of length L_0 = 3.485 m ($\omega_1/2\pi$ = 22 Hz) for frequencies: 1) ω/ω_1 = 0.41; 2) 0.46; 3) 0.47; 4) 0.50; 5) 0.52; 6) 0.55; 7) 0.64; 8) 0.69; 9) 0.73; 10) 0.87; 11) 0.96; 12) 1.0; 13) 1.2; 14) 1.28; 15) 1.37; 16) 1.42; 17) 1.46; 18) 1.50.

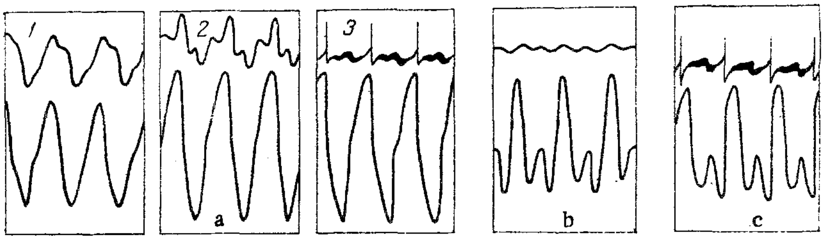

Fig. 2. Oscillograms of pressure (upper) and of velocity (lower curve) for the linear (a) ($1 - x/L_0$ = 0.861; 2 — 0.287; 3 — x/L_0 = 0; $\omega_1/2\pi$ = 22 Hz), for the first nonlinear (b) (x/L_0 = 0, $\omega_1^*/2\pi$ = 11 Hz), and for the second nonlinear resonance (c) (x/L_0 = 0, $\omega_2^*/2\pi$ = 33 Hz) in a pipe of length L_0 = 3.485 m.

the amplitude of the main velocity peak decreases, while that of the supplementary peak increases until the velocity fluctuation oscillograms lose their symmetric shape. The scale of the velocity fluctuations grows (frames 5, 6). In the pressure oscillograms the decrease in the main peak and the increase in the supplementary peak are at higher frequencies, so that the formation of a "saddle point" with equal peaks is completed only at frequency $\omega/\omega_1 \approx 0.55$ when the velocity oscillograms take on a symmetric shape (frame 6). Further increase in the piston frequency is accompanied by a decrease in the velocity, and a subsequent increase (frames 7-12) with the shape of the fluctuations being maintained. The symmetric shape of the pressure fluctuations appears at frequency ω/ω_1 = 0.87 (frame 10). Finally (frame 12), the first linear resonance sets in, with maxima in the velocity and pressure scales, and here the supplementary peak appears in the growth section. If one moves from the first linear resonance towards higher frequencies one finds that the symmetry is lost, there is a decrease in the main velocity peak, and the supplementary peak appears and grows in the velocity and pressure oscillograms. In contrast with the first nonlinear resonance, the saddle point in the pressure oscillogram at the frequency of the second nonlinear resonance ω/ω_1 = 1.5 (frame 18) is formed below. The maxima of the pressure and velocity scales are observed at the same frequency as that of the second nonlinear resonance (frame 18). In the frame corresponding to ω/ω_1 = 1.42, the saddle point with the same values of the velocity and pressure peaks points downwards, and here the velocity scale is a minimum.

Figure 2 shows oscillograms of velocity and pressure oscillations for the linear (a), the first nonlinear (b), and the second nonlinear resonances (c) in a pipe of length L_0 = 3.485 m. It can be seen that, for the linear and the second nonlinear resonances, shock waves are emitted from the open end of the pipe, while the pressure oscillogram has a symmetric and continuous shape for the first nonlinear resonance. Similar discontinuities may be seen also in the velocity oscillograms. It is not difficult to see, in addition, that the change in the symmetry and the appearance of the discontinuity are observed even close to the piston (frame 1, Fig. 2a).

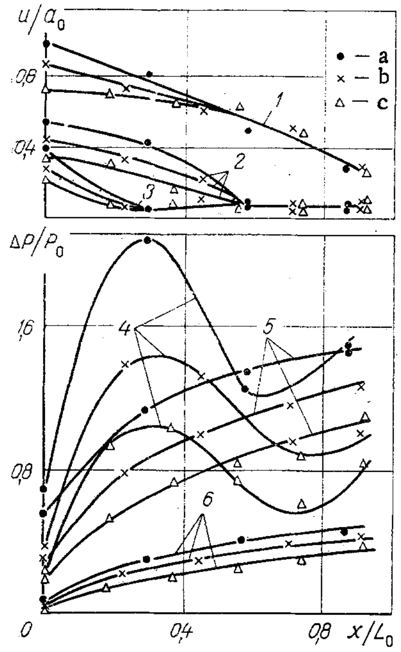

Fig. 3. Distribution of the dimensionless velocity scale u/a_0 (1-3) and the dimensionless pressure scale $\Delta P/P_0$ (4-6) along the pipe: a) L_0 = 3.485 m; b) 4.485; c) 5.485 m; 1, 5) linear resonance; 2, 6) first nonlinear resonance; 3, 4) second nonlinear resonance.

For the linear and the second nonlinear resonances, in addition to the peak, one also observes an intermediate discontinuity of smaller amplitude. Corresponding velocity discontinuities coincide in phase with the pressure fluctuations. The supplementary pressure peak appears even for the first nonlinear resonance, but it does not lead to the formation of a discontinuity.

Computation of the number n from the test data for the linear and the first nonlinear resonances shows that n = 0.5 in the system. This agrees with the data of [6-8] and is noticeably in contrast with other results [2, 3].

Figure 3 shows the distribution of the dimensionless scale of the velocity fluctuations and the dimensionless pressure scale for the linear (1, 5), the first nonlinear (2, 6), and the second nonlinear resonances (3, 4) for various pipe lengths.

The dimensionless scale of velocity fluctuations is a maximum at the open end of the pipe, for all the resonances; the scale decreases smoothly as one approaches the piston. The shorter the pipe the greater the velocity fluctuation scale. No influence of pipe length can be seen near the piston.

The dimensionless scale of the pressure fluctuations in the first nonlinear and linear resonances increases smoothly with increasing distance from the open end of the pipe. A change in pipe length causes a small change in the dimensionless pressure scale for the first nonlinear resonance. However, the influence of pipe length on the pressure scale is more significant for the linear resonance.

The pressure scale near the piston in the linear resonance is greater by approximately a factor of three than for the first nonlinear resonance. At the same time there is only a twofold increase in the dimensionless velocity fluctuation scale near the open end of the pipe.

The dependence of the dimensionless pressure fluctuation scale near the piston on the dimensionless velocity fluctuation scale at the open end in the linear resonance case can easily be correlated by the relation

$$\frac{\Delta P}{P_0} = \beta \frac{u}{a_0}, \tag{5}$$

where $\beta \cong 1.5$ for all pipe lengths. Using the theoretical results of [6, 7], one can establish a similar relationship between pressure at the piston and the velocity at the pipe exit with β = 1.4. The small difference in values of β lies within the experimental error.

Keller [12] has shown that for nonlinear resonances the amplitudes of the pressure and velocity fluctuations are complex functions of the piston motion amplitude l and of the length of the connecting rod driving the piston, as well as of the oscillation frequency. Since we

TABLE 1. Comparison of Calculated and Experimental Values of u/a_0 and $\Delta P/P_0$

Pipe length, m		3,485	4,485	5,485
$\frac{u}{a_0}$	Theory	0,48	0,42	0,36
	Experiment	0,52	0,39	0,32
$\frac{\Delta P}{P_0}$	Theory	0,48	0,42	0,37
	Experiment	0,52	0,40	0,28

do not have a formula for calculating the reduced length of the connecting rod, we can assume arbitrarily that the connecting rod length is calculated in the same way as the reduced amplitude of the piston displacement. Then the generalized parameter K, which has an appreciable influence on the amplitude of oscillations in the first nonlinear resonance, will vary in our conditions over the range 35-56. Table 1 shows estimates of the dimensionless fluctuations of pressure near the piston, and of velocity at the open end of the pipe, calculated from the Keller theory [12], and the corresponding experimental data.

For the second nonlinear resonance the dimensionless pressure fluctuation scale is distributed along the pipe like the second harmonic in a semiopen pipe, with an antinode of pressure near the piston, and a node of pressure and an antinode of velocity near the open end of the pipe. A decrease in pipe length causes an increase in the pressure scale.

It should be noted that the maximum in the dimensionless pressure fluctuation is larger by a factor of 1.5 than the pressure maximum in the linear resonance, which agrees with the results of [13], but indicates that the conditions at the open end have altered, i.e., one cannot regard the pipe as open.

The nature of the distribution of the dimensionless velocity scale shows that the velocity is a maximum near the open end, varies smoothly with distance from it, and reaches a minimum; then one can note a tendency to increase. No influence of pipe length on the velocity fluctuations was observed. In addition, the velocity at the open end is less by a factor of two than the dimensionless velocity scale in the linear resonance, and is on the same order as the velocity in the first nonlinear resonance. This circumstance is in poor agreement with the Keller theory [12], which indicates that the maximum velocity fluctuation scale must be larger, by a factor of at least 3 than the velocity fluctuations at the first nonlinear resonance.

Figure 4 shows the dimensionless pressure scale $\Delta P/P_0$ and the dimensionless velocity scale u/a_0 as a function of the dimensionless frequency ω/ω_1 for a pipe of length L_0 = 3.485 m ($\omega_1/2\pi$ = 22 Hz), at various distances from the open end. Resonances are observed on

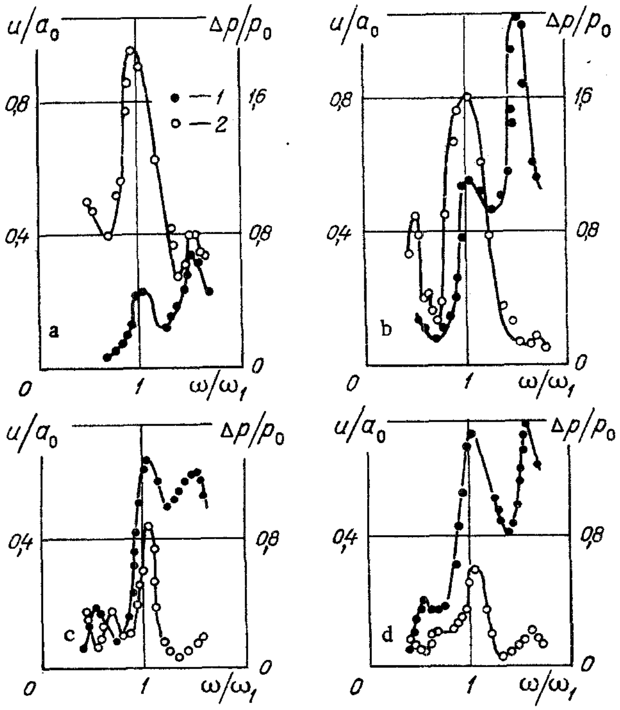

Fig. 4. Dimensionless pressure scale $\Delta P/P_0$ (1) and the dimensionless velocity scale u/a_0 (2) as a function of the dimensionless frequency ω/ω_1 in a pipe with L_0 = 3.485 m ($\omega_1/2\pi$ = 22 Hz): a) x/L_0 = 0; b) 0.287; c) 0.574; d) 861.

all the graphs, at frequencies $\omega = 0.5\omega_1$, ω_1, and $1.5\omega_1$, respectively. It is easy to see that the maximum pressure scale coincides in frequency with the maximum velocity scale only for the linear resonance. For the first nonlinear resonance the maximum pressure is reached at the minimum velocity (c, d). For the second nonlinear resonance the maximum pressure near the piston and at the open end coincide with the maximum velocity (a). Inside the pipe (b, c) this evidently does not hold.

Thus, by simultaneous measurement of velocity and pressure fluctuations over a wide frequency range, including the linear and nonlinear resonances, we have investigated the basic laws for resonances in open pipes.

It has been shown that in this system, for the linear and first nonlinear resonances, there is a power relationship with exponent 0.5 between the amplitude of the fluctuations and the pressure near the piston (or the velocity at the open end) and the amplitude of piston displacement.

For the linear resonance the amplitude of the pressure fluctuations near the piston and the amplitude of the velocity fluctuations at the open end are linearly related, and the first nonlinear resonance is described satisfactorily by the Keller theory.

NOTATION

l_0, piston stroke; R_0, piston diameter; R, internal diameter of the pipe; L_0, pipe length; h, height of conical adapter; a_0, speed of sound in the unperturbed gas; u, scale of velocity fluctuations; l, reduced amplitude of piston displacement; x, distance from the open end of the pipe; ω, excitation frequency; ω_1, ω_1^*, ω_2^*, self-frequencies for the linear, first nonlinear, and second nonlinear resonances; P_1, maximum in the pressure fluctuations; P_2, minimum in the pressure fluctuations; P_0, atmospheric pressure; $\Delta P/P_0 = (P_1 - P_2)/P_0$, dimensionless scale of pressure fluctuations; β, coefficient of proportionality; n, exponent; K, correlation parameter.

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EXPERIMENTAL STUDY OF THERMOMECHANICAL OSCILLATIONS OF A CYLINDRICAL HEATER IN AN AIR MEDIUM WITH FREE CONVECTION

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UDC 536.24

It is demonstrated experimentally that when appropriate conditions are fulfilled, oscillations in the heat liberation coefficient of an electrically heated wire vibrating in air are capable of parametrically exciting intense mechanical oscillations in the wire.

The effect of low-frequency oscillations of a heater on its heat liberation to the surrounding medium have been studied by a number of authors [1-3]. In [4-6] it was observed that in an electrically heated wire with a weight suspended from its midpoint nondamping transverse oscillations may develop under certain conditions. The authors explained this by the phenomenon of thermoparametric amplification.

The present authors have performed further studies of thermomechanical oscillations of a thin cylindrical heater, and of the role of heat exchange in the parametric resonance mentioned above.

We will enumerate the basic results obtained.

1. It has been shown experimentally that the presence of a suspended weight is not necessary for excitation of nondamping wire oscillations. Experiments were performed on a wire held rigidly at both ends ($l = 4.3$ m, $d = 4 \cdot 10^{-4}$ m) carrying dc current, the amplitude of which could be varied over a wide range. It was also possible to adjust the tension in the wire τ with a micrometer worm mechanism. The mean temperature T over the wire volume was determined from the wire's electrical resistance.

It developed that for each value of T there corresponded some interval of tension, in which wire oscillations were self-exciting. The amplitude of the oscillations A increased rapidly, reaching a limiting value A°, which was a function of the temperature difference ($\Delta T = T - T_0$) between the wire and the surrounding air: initially with increase in ΔT the

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